Modeling and Control of Membranes for Gossamer Spacecraft, Part 1: Theory

Marco B. Quadrelli, Samuel W. Sirlin
Jet Propulsion Laboratory
California Institute of Technology

August 4, 2000

Abstract

In this paper, Part 1 of 3, we derive the incremental equations of motion for a membrane to be used in simulations of gossamer spacecraft and, in particular, of precision inflatable structures. A boundary integral formulation is also presented, as a promising alternative to the finite element derivation. Part 2 of this report will show some numerical results obtained with the formulation outlined in Part 1. A discussion on control problems posed by large membrane structures in space will be the subject of Part 3.

Case 1

1 Introduction

The purpose of this paper is to shed some light on the dynamics and control problems one faces when modeling and analyzing gossamer-type spacecraft such as antennas built from inflatable structures, reflecting surfaces such as solar sails, and heat control surfaces such as solar shields. The term *gossamer* signifies *something light, delicate, insubstantial, or tenuous.* Examples are: a film of cobwebs floating in air; a large, ultra-lightweight system, packaged into a small launch volume. Typically, a gossamer spacecraft will be much (possibly orders of magnitude) lighter than a conventional structure, and will be constructed using composites and thin polymers. Its subsystems may be highly-integrated with a single light structure performing multiple functions. It may possess extensive adaptive capabilities, eventually capable of reconfiguring or evolving in response to changing mission conditions. Because of these attributes, gossamer systems offer breakthrough reductions in mission cost.

This work was performed at the Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, under contract with the National Aeronautics and Space Administration.

Gossamer spacecraft at the present moment in time may be classified into those used for large apertures, those used for solar sails, and those used for solar shields. All types present their own unique problems when it comes to modeling, simulation, and control. Inflatable structures have been proposed as a low cost alternative for large apertures in space. One of the problems that inflatable large apertures present is the necessity of reaching a high surface accuracy for the inflated membrane reflector, in order for the antenna to perform satisfactorily at the required electromagnetic bandwidth. For a radio-interferometric mission such as ARISE (Advanced Radio Interferometry between Space and Earth), the desired surface accuracy error on the 25 meter inflatable dish is below 1 mm rms for high frequency observations ($86GHz$). Figure 1 shows the ARISE inflatable antenna, and points out some of the fields in which technological advancements are necessary for the mission to become a reality.

The effective surface accuracy of an inflatable antenna depends on many factors such as: systematic manufacturing errors, long-term aging or creep of the polymeric membrane, quasi-static thermal distortions, and dynamic noise propagating from cooling equipment or attitude control devices. Even if in an ideal world most of these errors could be compensated by active means, there always remains a basic surface error. This error, the deviation from the desired surface, can be expressed as a sum of Zernike polynomials.

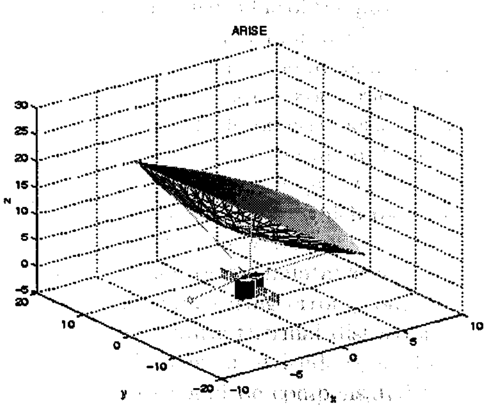

Figure 1. ARISE simulation model.

2 Nonlinear Membrane Modeling

Using a finite element method, we have derived the incremental equations of motion for a membrane to be used in simulations of precision inflatable structures. A boundary integral formulation has also been presented, as a promising alternative to the finite element derivation.

Some membrane models will be discussed. The idea to develop these membrane models came from the need to have a simulation tool that can deal with the dynamics of membranes of inflatable structures, and with the following characteristics: orthotropic or homogeneous linear elastic material, large displacements and small strain. The prototype problem for such a membrane is an inflatable reflector which, after inflation, assumes the form of a paraboloid. Some studies have already been done on the inflation of an otherwise flat membrane into a paraboloid [Ref.[1]]. It turns out that a membrane that inflates is always under a certain amount of pre-tension, which ensures that the surface is reasonably free of wrinkles. Details of wrinkle modeling or of the inflation procedure are not within the scope of this study and require further investigation.

Of interest in this report is the accuracy obtainable on the final shape of this surface after inflation, and also the method (or methods) to model and control this shape to a pre-specified accuracy. This is the essence of shape control.

Our modeling of an isotropic membrane subject to pressure, initial tension, and follower loads is based on the development of a displacement-based finite element interpolation scheme which includes the effects of initial tension, pressure load, and follower loads. Follower loads are included to represent actuator forces acting, to a first approximation, in the plane of the membrane. This is the case with embedded piezoelectrics. We obtain the total stiffness matrix for a membrane for space inflatable applications is given by the sum of the material stiffness matrix, the geometric stiffness matrix, the pressure stiffness matrix, and the follower load matrix. The total matrix is, in general, unsymmetric on account of the pressure and follower loads. The matrix is also dependent on the current state of deformation of the body.

3 Implementation

The Integrated Modeling of Optical Systems (IMOS) software [Ref.[2]], developed at JPL, is a Matlab computer program that provides an integrated environment for rapid thermal, structural, controls, and optical analysis of a system. IMOS currently covers only linear structural analysis, and so we extended some of its capabilities into the nonlinear range for membrane analysis.

Various options were considered for the membrane modeling, and were developed for IMOS to at least a theoretical level. Early efforts included a linear shell element; a linear membrane element with tension; a shell element with pre-stress. These elements are capable of providing accurate results in the small deformation regime, but may be used within a corotational formulation for large deformations. After these investigations we decided upon developing a nonlinear membrane shell element based upon previous work in the area of rubberlike models. The element uses nonlinear kinematics and a linear material constitutive equation and so is capable of modeling large deformations. Both three and six node triangular versions of the element have been coded into Matlab/IMOS. Using the Principle of Virtual Work, the residual and tangent stiffness are derived including effects of pre-stress, geometric nonlinearities, pressure, and follower loads.

4 Applications

Some applications of the include the dynamics and attitude control analysis of the ARISE spacecraft, and inflation and shape control of the ARISE reflector. Both applications use IMOS as primary tool.

4.1 Dynamics and Attitude Control Analysis

In the first case, a simple model of the ARISE inflatable antenna was built in IMOS. The simulation model of the ARISE spacecraft is shown in Figure 1. Half of the inflatable envelope is metallized to reflect both RF and sunlight. The lenticular envelope is supported by a torus, which is joined to a spacecraft bus by three struts. The torus and struts are to be inflatable, but later rigidized (e.g. by cooling past the glass transition temperature). The model is an off axis parabolic reflector, with scalable diameter, and focal length $f = D/2$. A simple circular mesh generator was added to IMOS to allow a set of plate or membrane elements to model the lenticular structure. Once the model was built, mode shapes were generated. Representative material properties were used for this $25m$ diameter case. A more detailed simulation using the membrane elements developed in Part 1 of this report will be described in Part 2.

4.2 Shape Control

Using a continuation method based on a homotopy approach, we have simulated the inflation process (Figure 2), and effects of boundary changes including control forces. Code has been developed to determine surface displacements upon application of the inflation pressure. Figure 3 shows the displacement of the torus when the surface is constrained to be on a paraboloid. More details on these numerical developments will be described in Part 2 of this report.

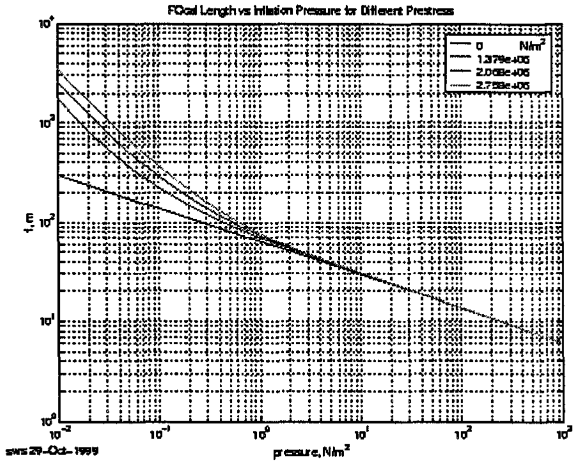

Figure 2. Focal Length vs. Inflation Pressure for Different Pre-stress levels.

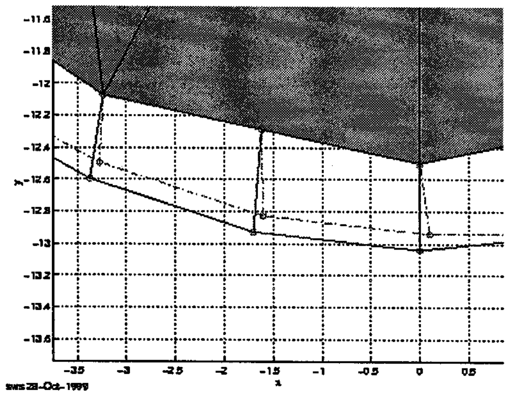

Figure 3. Displacement of torus and constant force springs under inflation pressure in order for the reflector to produce a parabolic surface.

5 Conclusions

The purpose of this paper is to shed some light on the dynamics and control problems one faces when modeling and analyzing gossamer-type spacecraft such as antennas built from inflatable structures, reflecting surfaces such as solar sails, and heat control surfaces such as solar shields. Using finite elements, we have derived the incremental equations of motion for a membrane to be used in simulations of precision inflatable structures. We show some numerical results obtained with the formulation here proposed, which extends JPL's software IMOS for applications with gossamer spacecraft.

References

[1] Campbell, J.D.: *On the Theory of Initially Tensioned Circular Membranes Subjected to Uniform Pressure*, Quarterly Journal of Mechanics and Applied Mathematics, vol. IX, part 1, 1956, pages 84-93.

[2] *Integrated Modeling of Optical Systems User's Manual*, Release 5.0, JPL Publication 98-12, Rev.A, February 2000.